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Tensor Network Theory of the Central Nervous System

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Tensor analysis is a mathematical discipline used to characterize physical quantities by generalized vectorial relations expressible in any particular system of coordinates. Tensor network theory of the central nervous system (CNS) is based on the abstract geometric concept that the brain internalizes relations existing in the external world by multidimensional vectorial relations, implemented within the CNS by neuronal networks. The theory describes in an abstract general manner, as well as in a particular quantitative fashion, how the brain may construct an internal representation of external relations of invariants by multidimensional vectors: the internal vectorial expressions are assigned to the external physical entities by the use of nonorthogonal intrinsic coordinate systems within the CNS.

The coordinate-system-free tensorial notation (better called coordinate-system-*general* representation) has already been successfully utilized in engineering and in the theory of relativity. In engineering, it was used to describe physical quantities such as distances, directions, forces, tensions (hence *tensor*), in a manner such that the choice of the particular system of coordinates used for a given vectorial representation is irrelevant. Since the realm of engineering is largely confined to classical mechanics, it uses tensors that are generally Cartesian [three-dimensional expressions in orthogonal frames], repre-

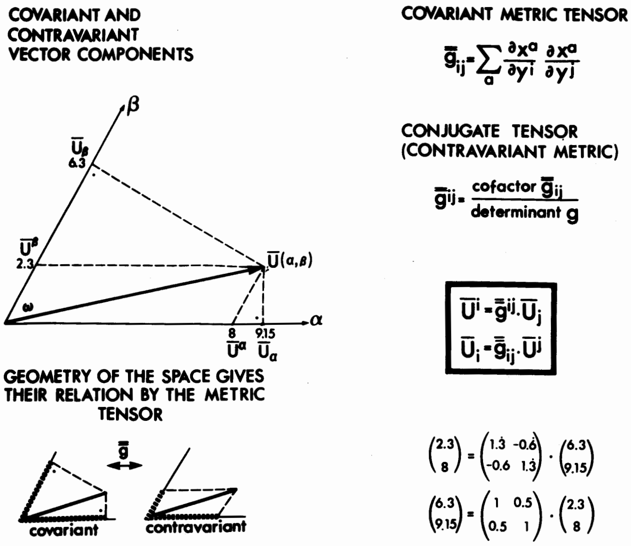

Figure 1. The fundamental distinction in tensor network theory of the CNS of covariant and contravariant vectorial expressions and their relationship through the metric tensor. From Pellionisz and Llinás (1980).

senting points in Euclidean space. In theory of relativity, which concerns relativistic mechanics, the geometry of the vector-space is Riemannian. Tensor network theory of the central nervous system necessitates a further generalization of tensor theory beyond Euclidean and Riemannian spaces since functional multidimensional spaces of the CNS are not necessarily limited to such well-known geometries.

One of the most fundamental mathematical requirements that are necessary in tensor network theory of brain function is the explicit distinction of covariant and contravariant versions of a vector, since the intrinsic natural systems of coordinates of the CNS are composed of axes that are usually not orthogonal to one another, and these forms are identical only in orthogonal frames of reference. These different vectorial versions correspond to the independently established but nonexecutional covariant vector components, yielding a sensory intention-type vector, and to the physically executable but interdependent contravariant vector components, yielding a motor execution vector. The relationship between covariant and contravariant vectorial versions is characterized by the metric tensor (see Fig. 1).

The covariant intention to contravariant execution transformation (via the contravariant metric tensor or, in case of singularity, the Moore-Penrose generalized inverse of the covariant metric tensor) can be described in abstract tensorial notation, as well as by a particular network transformation in any given frame of reference. Such a quantitative neuronal network model is shown in Figure 2, transforming vestibular sensory

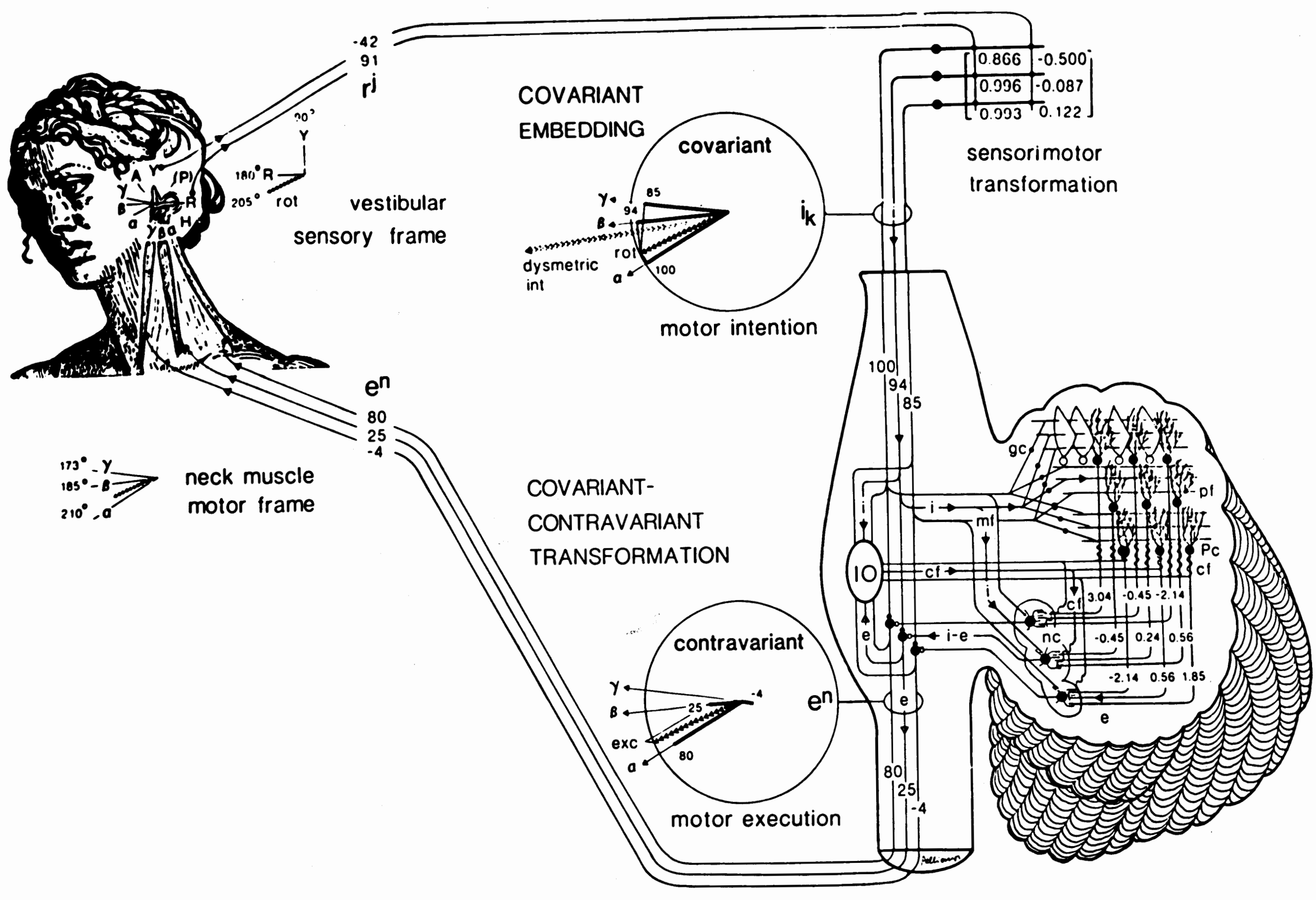

Figure 2. Tensor network model of the vestibulocollic reflex, embodying a covariant intention to contravariant motor execution transformation via the cerebellar neuronal network. From Pellionisz (1985).

coordinates into neck-muscle motor intention components and then through the cerebellum into motor execution vector components.

A physical entity, such as a head movement, is first expressed in a sensory system of coordinates, intrinsic to the organism, such as the vestibular semicircular canals. Then, a sensorimotor tensor transformation, called covariant embedding, yields projection-type covariant motor intention components, expressed in the neck-muscle frame of reference. The motor metric tensor-type transformation, converting covariant motor intention to contravariant motor execution by the Moore-Penrose generalized inverse of the covariant metric tensor, is performed by the cerebellar neuronal network, acting as a space-time metric tensor.

Beyond interpreting the general function of neuronal networks such as the cerebellum, tensor theory provides an explanation for the genesis and modification of neuronal networks. This hypothetical process, called metaorganization, predicts how physical and functional geometries may organize one another by means of reverberative oscillations of covariant proprioception and contravariant execution vectors. The process, implemented by physical tremors, can identify those special vectors, so-called *eigenvectors,* whose normalized covariant and contravariant versions are identical. If these fundamental functional vectors are imprinted into nuclear regions of the CNS, such as the inferior olive, they can govern the adaptive organization (genesis and modification) of higher order hierarchical structures such as the cerebellar corticonuclear neuronal network.

Experimental research has produced evidence that the central nervous system distributes activations of forelimb muscles in humans close to that predicted by the tensorial approach.

See also Cerebellum, Network Physiology; Integration, Neural; Motor Control

Further reading

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