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TENSOR NETWORK THEORY AND ITS APPLICATION IN COMPUTER MODELING OF THE METAORGANIZATION OF SENSORIMOTOR HIERARCHIES OF GAZE

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THE CHALLENGE

Neuronal networks are, in fact, used for "computations" in living organisms, producing what we call brain function (eg. sensorimotor coordination and intelligent representation). Neither the networks, nor their functions are fully known as yet, however. Based on what principle does the Central Nervous System (CNS) accomplish these tasks, and whether its mathematical understanding and subsequent or simultaneous technological implementation will lead to utilizable socioeconomical applications, are questions increasingly in the forefront of the interest of neuroscience community at large[1-3], and of its special field of brain theory, which is intimately tied to the artificial intelligence community and computer science and industry[4-7]. Activities range from mathematical analysis[8-11] to rehabilitation medicine[12,14]. The overlap of neuroscience with other disciplines created interdisciplinary subfields; Neurobotics[3,13,14], Neurophysics[16-18], and Neurophilosophy[19]. The new scientific revolution attracts neuroscientists spanning from molecular biologists[20,21] through mathematicians, engineers and physicists to philosophers. The implications warant an increasing awareness of their vital importance by government-agencies worldwide.

A GEOMETRICAL APPROACH TO BRAIN FUNCTION: TENSOR NETWORK THEORY

Motivated by the need of functionally interpreting the structure of existing neuronal networks[22], such as those in the *cerebellum*, this author strives for finding the basic general principle of the organization of "neuronal networks", and gainig a conceptual and formal grasp on what they "compute". The approach exposed here concentrates on sensorimotor neuronal networks (as in the cerebellum) and on the mathematical question of the axioms of their computations. Tensor network theory of the central nervous system may be summarized[1,2,14,15] by stating its axiom that the brain relates to the external world by expressing physical objects (invariants), both in a sensory and motor manner, in systems of coordinates that are *intrinsic* to the organism. Such general, typically non-orthogonal and overcomplete, frames of reference are physically obvious in sensory and motor parts of the CNS. Sensory and motor representation is identified in tensor network theory by *covariant vectors* [23] (with measurement-type orthogonal-projection components) and *contravariant vectors* (with physically executable parallelogram-type components), respectively. Thus, the *metric tensor operation*, which transforms these representations to one another was identified as a basic functional characteristics of sensorimotor networks eg. the cerebellum[29]

Beyond offering a formalism for describing neuronal computations of intrinsic vector-components of physical invariants, this approach conceptually features brain function as comprising functional geometries (via metric tensors, implemented by neuronal networks) in the internal CNS representation-spaces, both in sensorimotor and connected manifolds.

COMPUTER MODEL OF THE METAORGANIZATION OF GAZE SENSORIMOTOR HIERARCHIES

A quantitative example of this approach is a tensor model of gaze. To maintain a stable image in fixation, head & eye must compensate for passive movements or, in tracking, for the

AB.	NAME	YAW	PITCH	ROLL	AB.	NAME	YAW	PITCH	ROLL
(A)	**Retinal Ganglion Cells**				(D)	**Neck Muscle Motoneurons**			
MD	Medial direct.	-.155	.988	.000	LC	Longus Capitis	.129	.960	-.251
DR	Dorsal direct.	.988	.155	.000	CL	Cleidomastoid.	.047	.607	-.793
LT	Lateral direct.	.244	-.970	.000	ST	Sternomastoid.	.137	.590	-.796
VT	Ventral direct.	-.999	-.039	.000	LA	Longus Atlantis	-.395	.385	-.834
(B)	**Vestibular Canal Neurons**				QD	Dors. Sup. Obl.	.094	.318	-.943
HO	Horizontal	-1.000	.000	.000	QV	Ventr. Sup. Obl.	-.099	.287	-.953
AN	Anterior	.080	.693	.717	LI	Longiss.Capitis	-.290	.281	-.915
PO	Posterior	.080	-.693	.717	OQ	Obliq. Inferior	-.930	-.144	-.339
(C)	**Eye Muscle Motoneurons**				SP	Splenius	-.456	-.568	-.685
LR	Lateral Rectus	-.117	-.966	.232	CM	Complexus	-.133	-.571	-.810
MR	Medial Rectus	.001	.968	-.251	OC	Occipito-Scap.	-.321	-.693	-.645
SR	Super. Rectus	-.851	-.027	-.524	BV	Biventer	-.061	-.948	-.312
IR	Infer. Rectus	.891	.068	.449	RD	Rectus Medialis	-.263	-.958	.116
SO	Super. Oblique	.513	-.034	-.858	RN	Rectus Minor	-.218	-.963	.156
IO	Infer. Oblique	-.565	.158	.810	RM	Rectus Major	-.183	-.957	-.227

Table I. *Data, from computerized anatomy, to define coordinate systems intrinsic to tensorial expressions of gaze*. Rotational axes of (A): a mammalian retinal sensory frame[24], (B): frame of vestibular canals[25], (C): a motor frame of eye muscles[26], (D): neck-muscles[27].

movements of the target. In perfect gaze, the displacement and its compensation are identical; an example when *an identical invariant is expressed in various intrinsic coordinates.*

How various vectorial expressions within and among these frames are transformed by the CNS is the subject of tensor network theory: **A 3-step tensorial scheme** was elaborated to transfer covariant *sensory* vector to contravariants in a *motor* frame[2,28,32]

1) **Sensory metric tensor** (g^{pr}), transforming a covariant reception vector (s_r) to contravariant perception (s^p, lower and upper indices denote co- and contravariants):

$$s^p = g^{pr} \cdot s_r \quad \text{where} \quad g^{pr} = |g_{pr}|^{-1} = |\cos(\Omega_{pr})|^{-1}$$

where $|\cos(\Omega_{pr})|$ is the table of cosines of angles among sensory unit-vectors.

2) **Sensorimotor covariant embedding tensor** (c_{ip}) transforming the sensory vector (s^p) into covariant motor intention vector (m_i). Covariant embedding is a unique operation, regardless a dimensional inconsistency of the sensory and motor space (including over-completeness2), but results in a non-executable expression[23]:

$$m_i = c_{ip} \,.\, s^p \quad \text{where} \quad c_{ip} = u_i \,.\, w_p \quad \text{where}$$

u_i and w_p are the i-th sensory unit-vector and p-th motor unit-vector.

3) **Motor metric tensor**[1,2,23] that converts intention m_i to executable contravariants; $m^e = g^{ei} \,.\, m_i$ (where g^{ei} is computed as g^{pr} was for sensory axes in 1).

In case of overcompleteness, of either or both sensory and motor coordinate systems (as in **A,C,D** in Fig. 1), tensor network theory hypothesizes[23] that the CNS uses the **Moore-Penrose generalized inverse** (**MP**) of the unique covariant metric[2,15,30]:

$$g^{j,k} = \sum_m \{1/L_m^+ \,.\, |E_m > < E_m|\}, \quad \text{where}$$

E_m and L_m^+ are the m-th Eigenvector of $g_{j,k}$ and its Eigenvalue (replaced by 1 if it was 0).

This 3-step scheme is used to compute tensors of a sensorimotor reflex[2,28]. For the 4 gaze reflexes of **Fig.1**, *each expressing an invariant both in a sensory and a motor frame*, the above calculation yields tensor-matrices as shown (by patch-diagrams only) in **Fig.2.**

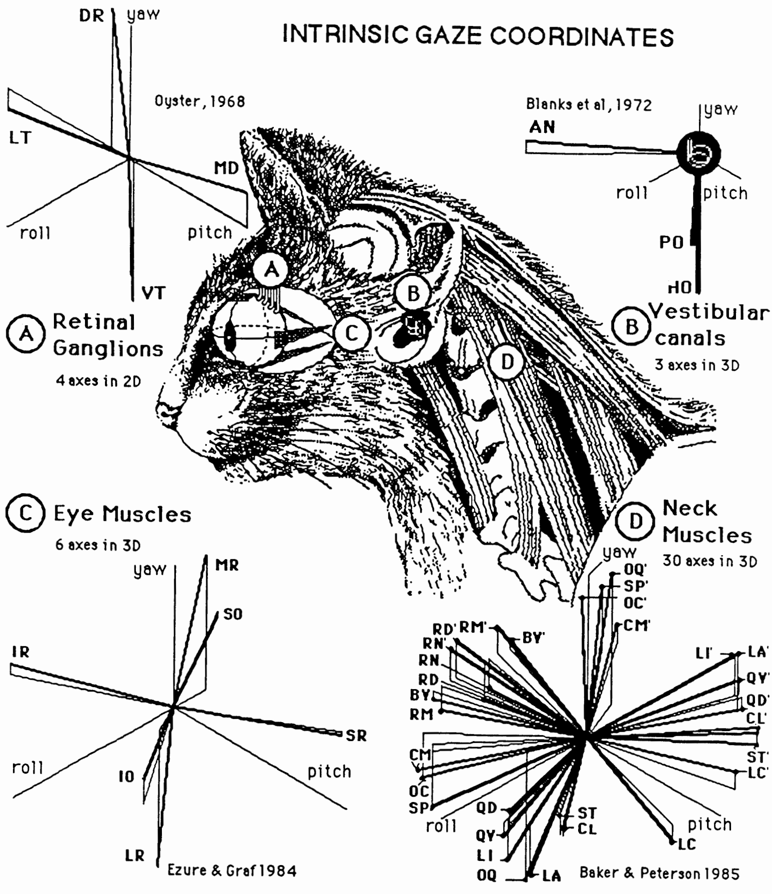

Fig. 1. *Coordinates intrinsic to gaze sensorimotor neuronal networks.* Gaze is expressed by rotations of the head and eye via the neck and eye muscles, so that they compensate for rotations measured by the retinal ganglion cells and by the vestibular semicircular canals. Both the dual (retinal and vestibular) sensory apparatus and the dual (oculomotor and neck-motor) executor systems operate along rotational axes determined by the structure of the organism.In order to express gaze, neuronal networks must measure and produce physical invariants (movements) in these typically non-orthogonal, overcomplete intrinsic frames of reference, by covariant sensory and contravariant motor vectors. Since the frames have been established by quantitative anatomy (cf. Table I.), the problem that we face is to quantitatively interpret how the CNS establishes relationships among various vectorial representations rendered to of a physical invariant such as gaze-displacement. *Tensor tranformations* yield a general interpretation as well as a means of calculation by tensor-matrices, implemented in the CNS by the system of interconnections in *neuronal networks* .

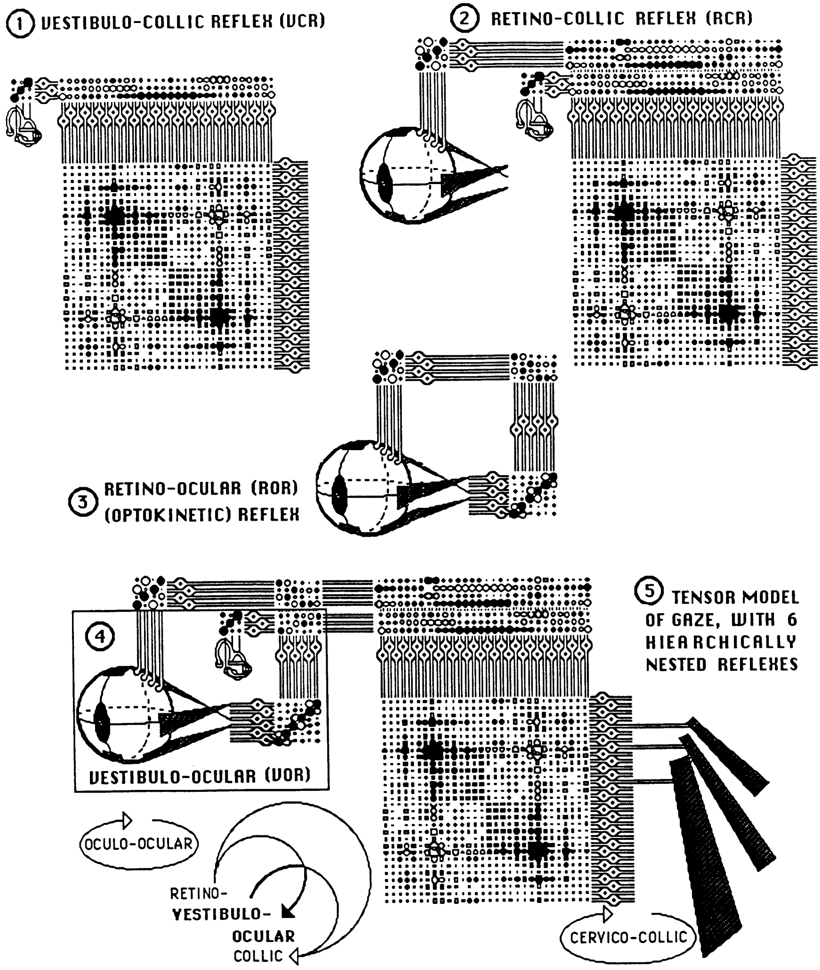

Fig.2. *Metaorganization, in six developmental steps, of sensorimotor reflexes of gaze.* Three neuronal networks, required for tensor-transformations in *each* sensorimotor reflex, eg. from vestibular- to neck-motor vector in (1), were calculated[2,28,31] by the 3-step tensor scheme. Resulting tensor-matrices are shown by three patch-diagrams in each sensorimotor reflex-arc; VCR(1), RCR(2), ROR(3), VOR(4). These networks are to develop in a definite sequence; in the VCR(1) the motor metric, sensorimotor embedding and sensory metric develop, as described by metaorganization[31]. RCR(2) builds hierarchically on the existing neck-motor metric, and the retinal metric is used also for ROR(3). VOR(4) is built on top of this hierarchy, using the vestibular metric available from VCR. Since the VOR is the only gaze reflex which is *not* a closed-loop sensorimotor system[28], its development must use the already available RCR,VCR,ROR networks. (The oculo-ocular & cervico-collic motor metrics whose development *preceed* those of the 4 gaze reflexes are also indicated in scheme 5).

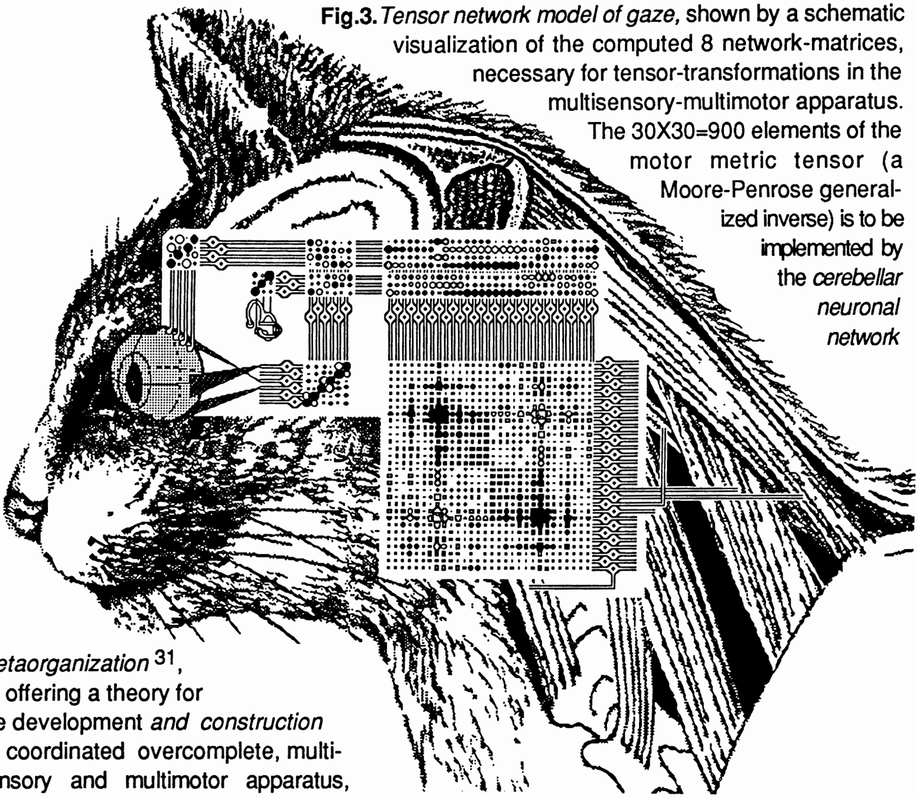

Fig.3. *Tensor network model of gaze,* shown by a schematic visualization of the computed 8 network-matrices, necessary for tensor-transformations in the multisensory-multimotor apparatus. The 30X30=900 elements of the motor metric tensor (a Moore-Penrose generalized inverse) is to be implemented by the *cerebellar neuronal network*

Metaorganization [31], by offering a theory for the development *and construction* of coordinated overcomplete, multisensory and multimotor apparatus, appears to yield both mathematical and neurobiological advantages.

As for theory and implementation, advantages result from its employing the MP formula, which a) yields the proper inverse if the space is complete, b) yields a least-squares minimum-energy formula, c) can be generated by the CNS via the process of metaorganization (also yielding the sensory metric and sensorimotor covariant embedding networks), by the utilization of reverberative oscillations[31], d) the theoretical prediction *has been experimentally shown* to conform with the CNS in gaze control[32] and in coordination of human arms[33].

As for Neuroscience, tensor network theory may be useful by functionally interpreting existing neuronal networks. As suggested, the proposed metric-type function can be implemented for sensory modalities by the tectum[30], for motor vectors by the cerebellum[29]. Structural features of the proposed tensor-transformation matrices in sensorimotor reflexes, eg. the three tensor-transformations in the VCR and VOR, appear to match structural properties of the CNS, where eg. vestibular signals are known to be transformed from the semicircular canals to vestibular nuclei, from there to premotor nuclei, and to oculomotor nuclei, before reaching eye muscles[28,32]. While it may require an enormous cooperative effort, quantiative anatomy, experimental network analysis and tensor network theory may gradually reveal not only quantitative operational features of neuronal circuitries, but also basic principles of brain function. This work starts on the proving ground of Brain Theory, sensorimotor coordination, where the physical entities that are the objectives of neuronal computation are most evident. Principles and techniques learnt from these studies, if truly general, could be helpful for understanding intelligent representations in the neocortex.

FUNCTIONAL GEOMETRIES IN CONNECTED CNS REPRESENTATION HYPERSPACES

Metaorganization of gaze networks is an example for creating sensory- and motor metric-type networks. They comprise functional geometries to match the physical geometry of the structure of sensory and motor apparatus. Neuronal networks in the brain, however, incorporate functional geometries that not only passively react to given physical geometries, by compensating for modifications occurring in their relation to the environment as in gaze, but *impose intelligent function* on both the sensorimotor apparatus, and ultimately on the world[15]. Such active modification, to be intelligent, requires the brain to comprise a functional geometry, *a world model, with a geometry that is homeomorphic.* Intelligent functional geometries of the CNS are presently largely unexplored, but are expected to transgress the boundaries of Euclidean or Riemannian manifolds[15]. Thus, a study of sensorimotor spaces that are directly connected to known external structural geometries may be essential homework. Then the generalized principle of metaorganization and the general formalism of tensor network theory may be applied to tackle the ultimate question of how intelligent geometries develop in the CNS, or can be developed to extend them.

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