Nature often provides inherently oblique systems of coordinates, where Cartesian conventions are not valid. The extraocular motor system is an example of a geometry in which the intrinsic frame of reference is clearly non-orthogonal. Previous models (Robinson, Boeder) have defined the positional parameters of the extraocular muscles. It is apparent that a general approach to coordinate systems, such as tensor analysis, is needed. Expressing the above models with this method, applicable to any coordinate system, the implications of nonorthogonality and an overcomplete number of axes can be revealed naturally.

While an eye movement is a physical invariant, we describe it vectorially in "oculomotor hyperspace" by either covariant or contravariant components. By definition, movement arises through the physical summation of contravariant vector components. However, if the CNS were to relay intention (covariant) vectors directly to the extraocular muscles, the degree of accuracy in the production of intended movements would be a measure of the orthogonality of the system. For saccadic eye movements we work with two coordinate systems, one based upon the actual length changes of the six extraocular muscles and their axes of rotation, and another which is a projection of the first system onto a tangent plane. We characterize saccadic eye movements (from primary, secondary, and tertiary positions) by both covariant and contravariant components. Allowing covariant vectors to be relayed to the muscles (as if they were contravariants), this movement will differ from the original direction by an "error" angle $\omega$, as shown.

Orthogonality would result in eigenvectors (defined by $\omega=0$) in all directions. Our results show that the execution of covariants leads to an error in every direction except for the four eigenvectors of the system. Supported by Grant NS13742.
Tensor network theory applied to the oculomotor system. CNS activity expressed with natural, non-orthogonal coordinates.


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