[40] Pellionisz, A. and Llinas, R. (1979) Cerebellar Coordination: Covariant Analysis and Contravariant Synthesis via Metric Tensor. A Tensorial Approach to the Geometry of Brain Function. Soc. Neurosci. Abst. 5.105

342 CEREBELLAR COORDINATION : COVARIANT ANALYSIS AND CONTRAVARIANT SYNTHESIS VIA METRIC TENSOR. A TENSORIAL APPROACH TO THE GEOMETRY OF BRAIN FUNCTION. A. Pellionisz and R. Llinás. Dept. Physiol. & Biophys., New York Univ. Med. Ctr. 550 First Ave, New York 10016

A wealth of experimental data indicates brain function (e.g. motor coordination by cerebellar networks) to be a distributed and parallel property. It is imperative, however, to develop formal treatments capable of allowing precise conceptual descriptions and quantitative formulations of such global properties. Thus, a tensorial approach was introduced (Pellionisz & Llinás: *Neurosci. Abst.* 4,1978; *Neuroscience* 4:323,1979) which considered brains in terms of abstract geometry. Here, tensor network theory is utilized in the analysis of the coordination of limb movements and vestibulo-ocular responses.

Movements emerge from collective vectorial actions of the many segments of a limb, or of the extraocular muscles. The present treatment is based on the view that movements are reference frame invariant vectors (i.e. tensorial entities). Since movement vectors occur both in the 3-dimensional space and in the multi-dimensional space of the CNS, the fundamental problem of motor coordination is: how can an intended movement (which refers to the 3-space) be executed by a high dimensional motor system? This leads directly to the inherent properties of the spaces that contain the vectors : *a) Is the CNS hyperspace metric?* Then, if the coordination problem is regarded as an embedding of the 3-dimensional space into the CNS hyperspace, the question is: *b) How can the decomposition be unique despite the overcompleteness of the hyperspace?*

We assume that the CNS space *is* endowed with an inherent geometry: which in the case of motor system is given by the prewired matrix of the cerebellar metric tensor: Θ. Assuming a local homomorphism of the two overlapping spaces (3-dimensional and CNS hyperspace) leads to a two-step scheme of coordination. *(1) A decomposition of the intended vector into an overcomplete number of covariant components, using the geometry of the three-space, (2) A transformation of the covariant components into contravariant components by a CNS metric tensor.*

As shown by computer modeling, the covariant vector components can be established, even for an overcomplete number of coordinates. However, when used directly to generate the intended movement, the covariant components yield an ataxic, dysmetric movement, but when transformed to contravariant components, the intended movement is generated in a coordinated unique style.

Beyond providing a formal scheme of coordinated motor action by the known cerebellar neuronal network, some general conclusions are reached regarding the properties of CNS space and generalizing the covariant analysis and contravariant synthesis to sensory systems. (Supported by USPHS grant NS13742 from NINCDS)

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